Voronoi Diagrams in L_∞ - for VLSI Critical Area Computation

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Voronoi Diagrams are important in VLSI CAD

There are different variants of Voronoi Diagram such as nearest-neighbor, higher order and the farthest-neighbor which fits in naturally in different practical situations of a VLSI layout.

- The critical Area extraction for shorts in a VLSI layout can be computed as a function of the second order L_∞ Voronoi diagram.
- The critical area for opens can be modeled using higher order Voronoi diagrams.
- The problem of computing critical area for via-blocks reduces to computing a Hausdorff Voronoi digram of polygons representing clusters of redundant vias.

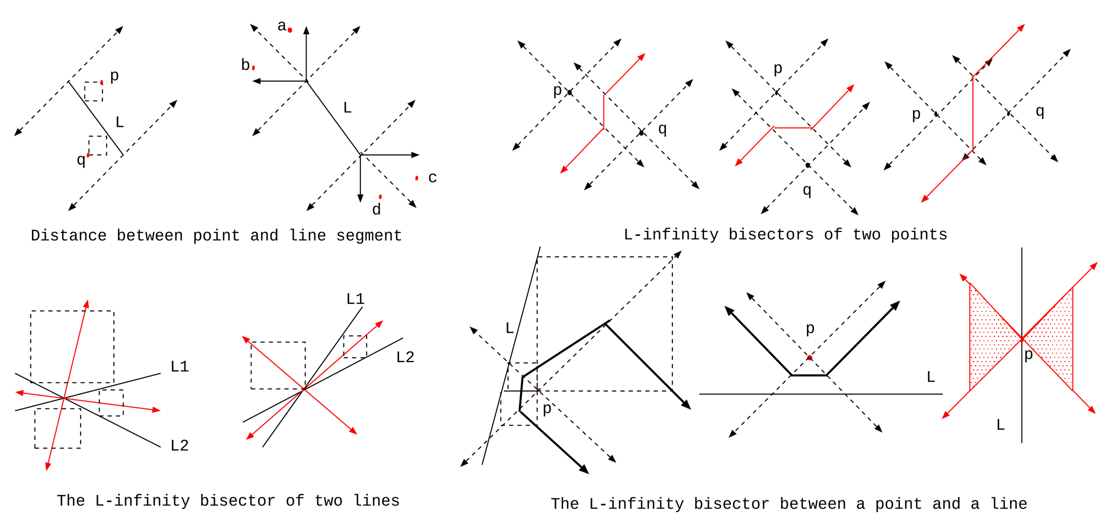
The extraction of critical area is a fundamental problem in VLSI yield prediction.

Why and When L_∞?

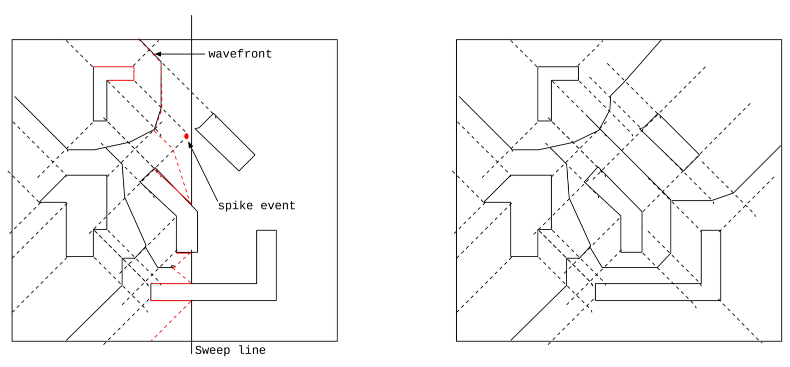
- Parabolic arcs in Euclidean Voronoi diagram make its robust implementation harder. L_∞ Voronoi diagram contains only straight line segments.
- In applications where Euclidean accuracy is not needed, L_∞ can give a more practical and simple solution.

L_∞ Distance and Bisectors

- The L_∞ distance between the two geometric objects is the side length of the smallest isothetic square touching them.



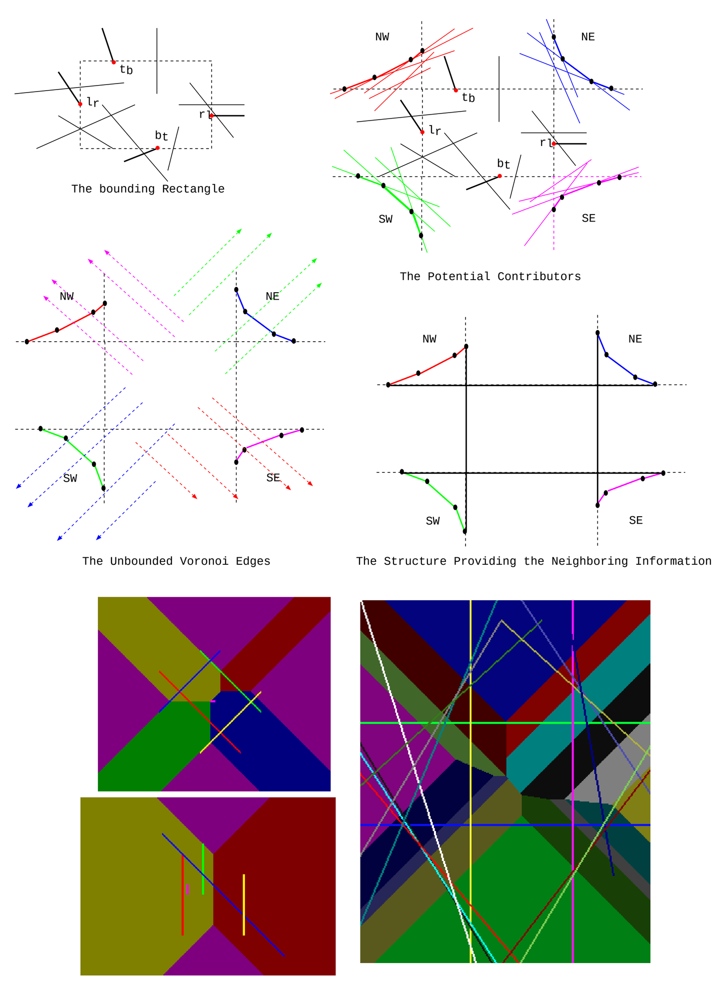
L_∞ Voronoi Diagram of Polygons



Data-Structures required to construct and store the Voronoi diagram:

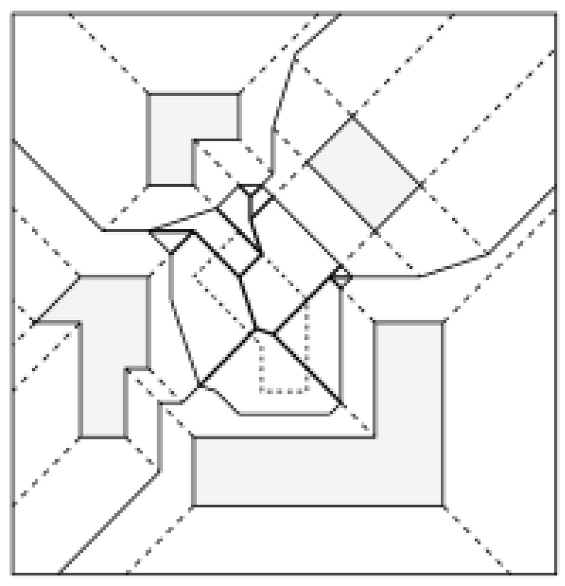
- A Priority Queue to store the events.
- A balanced binary tree to store the sweep line status.
- A half edge data-structure (DCEL) to store the planer subdivision.

L_∞ Farthest Segment Voronoi Drigram



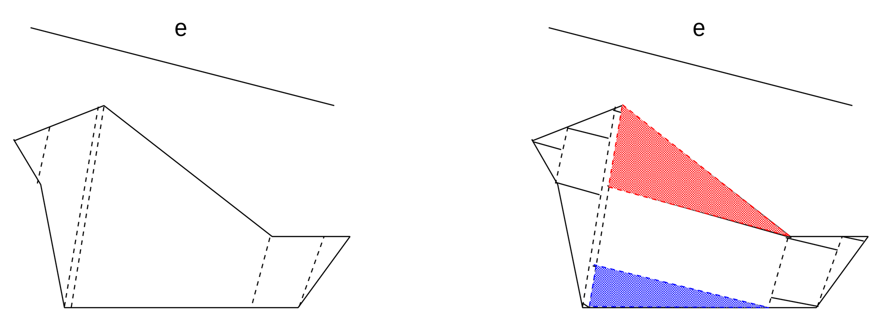
- Given the structure of neighborhood information the farthest segment voronoi diagram in L_∞ can be computed in linear time.

2^{nd} order subdivision with in a Voronoi cell



Critical Area for Shorts

- The *critical radius* of an arbitrary point p in the layout for shorts is the radius of the smallest defect which if centered at p overlaps with two different polygons.
- The critical radius of p is the distance of p from the second nearest polygon to p.
- Critical area within a second order Voronoi cell V is given by: $A_c(V) = \int_0^\infty A(r,V)D(r)dr$, which can be discretized as a summation ($A_c = \sum_V A_c(V)$) of second order L_∞ Voronoi edges.



The decomposition of a second order Voronoi cell V into trapezoids

- The critical area within a rectangle R is $A_c(R) = \frac{r_0^2 S}{2}(\frac{l}{r_j} - \frac{l}{r_k})$
- The critical area within a red triangle T_{red} is given by $A_c(T_{red}) = \frac{r_0^2 S}{2}(STln(\frac{r_k}{r_j}) - \frac{l}{r_k})$
- The critical area within a blue triangle is given by T_{blue} is $A_c(T_{blue}) = \frac{r_0^2 S}{2}(\frac{l}{r_j} - STln(\frac{r_k}{r_j}))$

  l is the size of the edge of R, T_{red} and T_{blue} parallel to e. r_j and r_k minimum and maximum critical radius of their vertices.

- Given $2nd$ order Voronoi diagram of polygons oa a layer in a VLSI circuit and assuming that the defects are square following the r_0^2/r^3 defect density distribution, the critical area for shorts in that layer is given by

$$A_c = r_0^2(\sum_{\substack{red,\\prime\\e_i}} \frac{S_i l_i}{r_i} - \sum_{\substack{blue,\\prime\\e_i}} \frac{S_i l_i}{r_i} + \frac{1}{2}\sum_{\substack{red,\\nonprime\\e_j,\\wrt\\e}} S_e^2 T_e ln\frac{r_k}{r_j} - \frac{1}{2}\sum_{\substack{blue,\\nonprime\\e_j,\\wrt\\e}} S_e^2 T_e ln\frac{r_k}{r_j} + \frac{B}{2})$$

Discussion

The L_∞ Voronoi diagram is a simple planar subdivision consisting of straight line segments and is perticularly useful in the VLSI CAD applications where proximity information is needed.

References

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